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Space environment (natural and artificial) — Geomagnetic reference models

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Foreword

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This ISO Guide was prepared by Technical Committee ISO/TC 20, *Aircraft and space vehicles*, Subcommittee 14, *Space systems and operations*.

Introduction

For centuries, geomagnetic reference models have been used to describe the vector field (for example by its direction and strength) as a function of position and date. Such models are widely used in upper-atmospheric, ionospheric and magnetospheric research, and in characterizing the near-Earth space environment. They further provide an essential reference for navigation, heading and attitude determination and control subsystems of spacecraft and ground based systems.

Earth’s magnetic field is represented in such models as a spherical harmonic expansion of the equivalent scalar magnetic potential. This representation was proposed by Karl Friedrich Gauss (1777-1855) and has been used ever since to describe the geomagnetic field. The spherical harmonic coefficients of the geomagnetic field are commonly called ‘Gauss coefficients’. In 1969 the International Association for Geomagnetism and Aeronomy (IAGA) introduced the International Geomagnetic Reference Field (IGRF) which uses the Gauss representation to describe Earth’s magnetic field. This International Standard closely mirrors the established specification of such models, including equations and computational procedure.

There are several internal and external sources contributing to the observed magnetic field. All of these sources affect a scientific or navigational instrument, but only some of them are represented in geomagnetic reference models. The strongest contribution, by far, is the magnetic field produced by motions in the Earth’s liquid-iron outer core; this is called the core field. This core field changes perceptibly from year to year. When extrapolating the temporal evolution of the field into the future, a linear extrapolation of the Gauss coefficients is used. Geomagnetic reference models specify the Gauss coefficients for a start date (epoch) and provide their linear change over time as a set of so-called secular variation (SV) coefficients. Due to unpredictable non-linear changes in the core field, predictive geomagnetic reference models are valid only for a limited period, and users subsequently have to update to a newer version.

Other sources also contribute to the magnetic field: Magnetic minerals in the crust and upper mantle give rise to magnetic anomalies which can be significant locally. Electric currents induced by the flow of conducting sea water through the ambient magnetic field make a further, albeit weak, contribution to the observed magnetic field. Time-varying electric currents in the upper atmosphere and near-Earth space generate an external magnetic field. The external field does not average to zero over time. Its steady contribution can therefore be included in a geomagnetic reference model using external Gauss coefficients with linear secular variation. Time varying external magnetic fields further induce electric currents in the Earth and oceans, producing secondary internal magnetic fields. Since there is no general consensus on how to separate these various internal and external sources, it is left to the producer of a geomagnetic reference model to specify which of these internal and external sources are included in their model, and any radial limitation to the validity of the external part of their model

Space environment (natural and artificial) — Geomagnetic reference models

1 Scope

This International Standard defines reference models representing the geomagnetic field. It closely mirrors and clarifies specifications which have been in use for many decades.

The approach is to represent the corresponding scalar magnetic potential by a spherical harmonic expansion having specified numerical coefficients, called Gauss coefficients. Changes of the magnetic field are modelled by a linear time-dependence of each Gauss coefficient. This document provides the equations and a step-by-step computational procedure to evaluate a geomagnetic reference model for any desired location and date.

Not specified in this standard is the interpretation of the geophysical content of a geomagnetic reference model. It is left to the producer of a model to specify which internal and external magnetic sources are included in (or excluded from) their model.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

NIST Handbook of Mathematical Functions (ISBN 978-0-521-14063-8, paperback), on-line at http://dlmf.nist.gov/14.

Backus, G., R.L. Parker, and C. Constable, 1996. *Foundations of geomagnetism*. Cambridge University Press.

Gradshteyn, I.S. and I.M. Ryzhik, 1994. *Table of integrals, series and products* (5th ed). Academic Press.

Heiskanen, W. and H. Moritz, 1967. *Physical geodesy*. San Francisco: W.H. Freeman and Company.

ISO 19111:2007, *Geographic information — Spatial referencing by coordinates*

ISO 22009, *Space systems — Model of the Earth's magnetospheric magnetic field*

Langel, R.A., 1987. The main field. In *Geomagnetism*, edited by J.A. Jacobs, Academic Press, 249-512.

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1 Reference frames

For positions remote from the Earth it is customary to use a geocentric reference frame, and to resolve the magnetic field vector into components based on this geocentric frame. For positions on and near the Earth's surface it is customary to use a geodetic reference frame, based on the standard WGS84 ellipsoid of rotation approximating the Earth's surface, and to resolve the magnetic field vector into components based on this geodetic frame. Note that the down axis in the geodetic frame is perpendicular to the surface of the WGS84 ellipsoid. Deflections of the vertical caused by local gravity anomalies have to be taken care of by the user and are not covered by this standard (see Section 4.2). Geocentric and geodetic coordinates are illustrated in Figure 1.

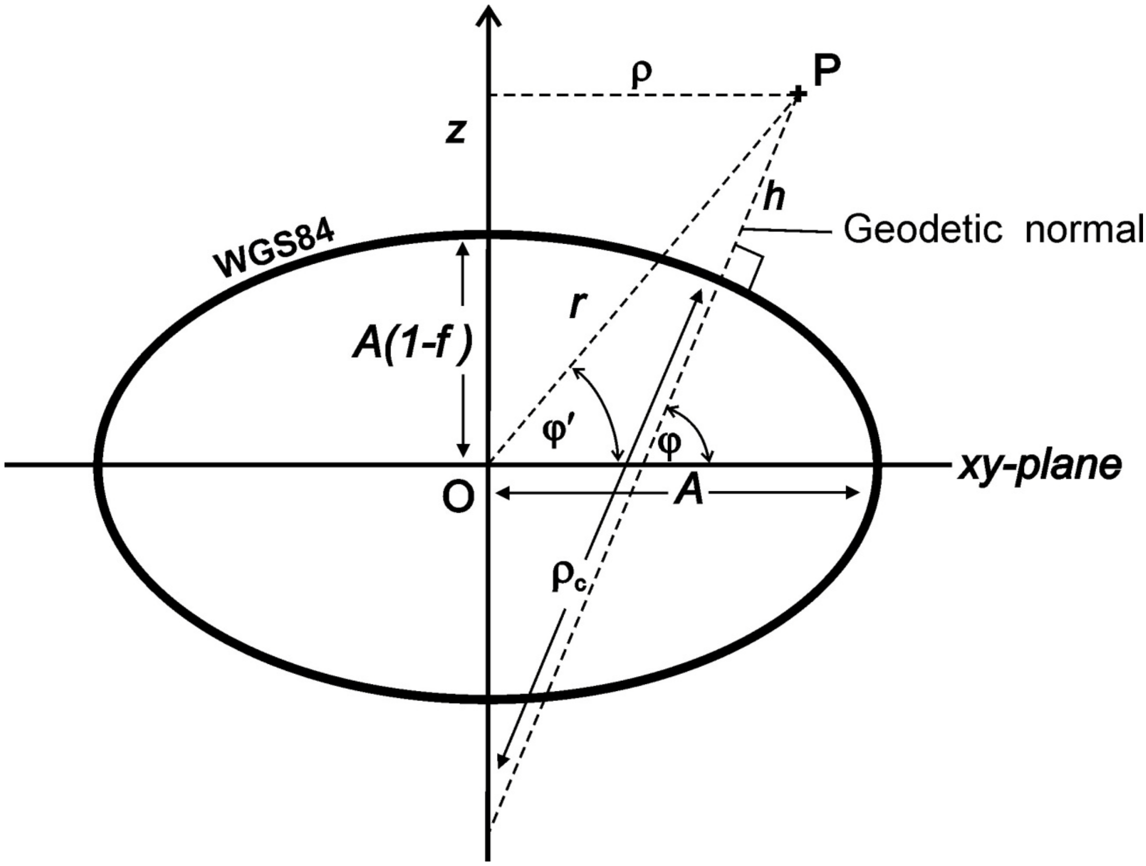

Figure 1: An axial cross-section through the point of interest P which is at longitude λ.This point P is specified in geocentric spherical polar coordinates by its distance *r* from the Earth center O and the incidence angle of the OP line with the equatorial xy-plane. The same point P has geodetic coordinates given by its height *h* above the WGS84 reference ellipsoid and the incidence angle of its geodetic normal with the xy-plane.

3.1.1 Geocentric reference frame

A point location in geocentric Cartesian coordinates is given by the same (x, y, z) as are used in geodetic Cartesian coordinates – see Section 3.1.2 (but note that ISO 19111 uses capital X, Y, and Z). Equivalently, coordinates can be specified as geocentric spherical polar (λ, φ´, *r*), where *λ* is longitude, φ' is geocentric latitude, and *r* is distance from the Earth center. The prime is used to distinguish geocentric from geodetic terms where necessary.

3.1.2 Geodetic reference frame

The geodetic reference frame is based on the WGS 84 reference ellipsoid. This is an ellipsoid of rotation having a defined semi-major axis (in the equatorial plane) *A*, and a flattening *f*. This leads to a semi-minor axis (essentially along the spin axis) of *A(1-f)*. Specifically

$$A = 6378137\,\text{m}$$

$$\frac{1}{f} = 298.257223563 \tag{1}$$

$$e^2 = f(2 - f)$$

where *e* is the (first) eccentricity.

A point location in geodetic coordinates is given by (λ, φ, h), where λ is longitude, φ is geodetic latitude, and h is the height (distance normal to the ellipsoid) above the WGS 84 reference ellipsoid. Geodetic and geocentric longitudes are identical. This nomenclature follows ISO 19111. A point location can also be specified by the Cartesian coordinates (x, y, z) in the WGS84 reference system, where the positive z and x axes point in the directions of the semi-minor axis and the prime meridian ($\lambda=0$) respectively; this is the same as the geocentric Cartesian coordinate system. For the WGS 84 ellipsoid we then have for ρ_c, the radius of curvature of the normal section at the geodetic latitude φ,

$$\rho_c = \frac{A}{\sqrt{1-e^2 \sin^2 \varphi}} \tag{2}$$

3.1.3 Geodetic to geocentric coordinate transform

The geodetic coordinates (λ, φ, h) are transformed into spherical geocentric coordinates (λ , φ´, r) by recognizing that λ is the same in both coordinate systems, and that (φ r) is computed from (φ, h) according to the equations:

$$\begin{aligned} \rho &= \sqrt{(x^2 + y^{2)}} = (\rho_c + h)\cos\varphi \\ z &= (\rho_c(1-e^2) + h)\sin\varphi \\ r &= \sqrt{\rho^2 + z^2} \\ \varphi' &= \arcsin\frac{z}{r} \end{aligned} \tag{3}$$

Here, ρ is the east-west (cylindrical) radius of curvature.

3.2 Specification of the magnetic vector B

3.2.1 Magnetic vector components in the geocentric frame

At the point of interest P(λ, φ´, r), the geomagnetic field vector **B** can be described by 3 orthogonal components in a local Cartesian coordinate system with origin at P and axes in the geocentric spherical-polar directions given by dλ, dφ´ and dr. For historical reasons, however, the triplet (X', Y', Z') is used (Fig. 2), where X' is the northerly intensity, Y' the easterly intensity, and Z' the inward radial intensity, positive towards the Earth's center. The unit vectors of the geomagnetic field components X', Y', and Z' thus point in the φ´, λ, and negative-r directions, respectively. The quantities X', Y', and Z' are the sizes of perpendicular vectors that add vectorially to **B**. Note that the orientation of this coordinate system varies with angular position. The synthesis of the field from a spherical harmonic model initially produces these X', Y'. and Z' components.

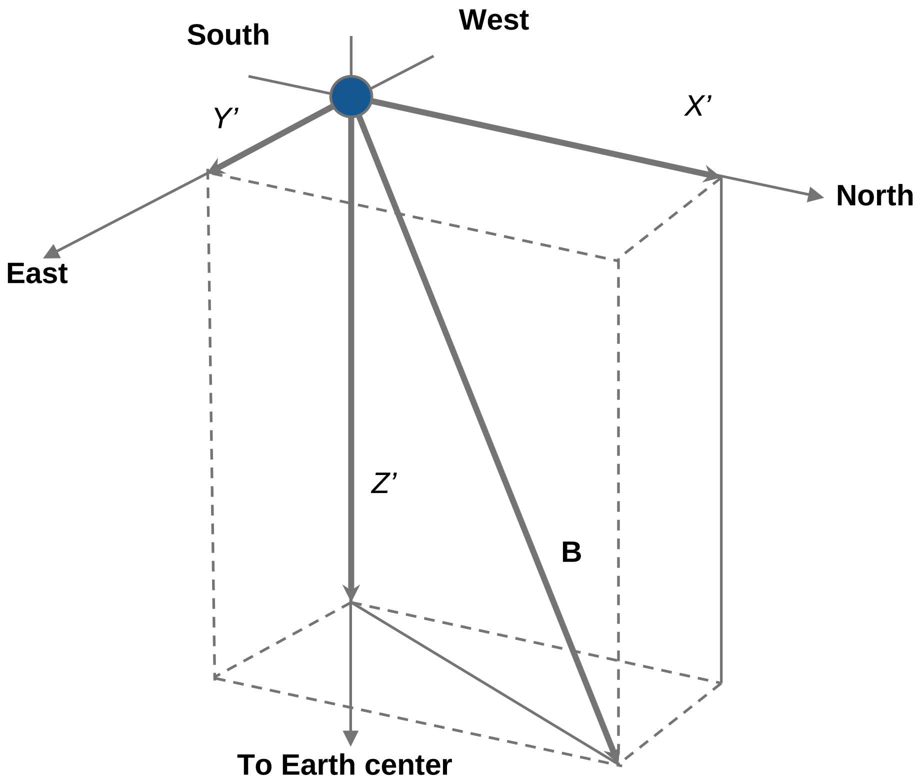

Figure 2: The *X', Y'* and *Z'* components of the geomagnetic field vector **B** in the geocentric reference frame

3.2.2 Magnetic elements in the geodetic reference frame

The geomagnetic field vector, **B**, is fully described by an appropriate set of three elements, but in practice 7 elements are variously used (see Fig. 3). The main orthogonal set is the northerly intensity *X*, the easterly intensity *Y*, and the vertical intensity *Z* (positive downwards). Another orthogonal set is the total intensity *F*, the inclination angle *I*, (also called the dip angle and measured from the horizontal plane to the field vector, positive downwards), and the declination angle *D* (also called the magnetic variation and measured clockwise from true north to the horizontal component of the field vector). The seventh element is the horizontal intensity H. In the descriptions of *X, Y, Z, F, H, I* and *D* above, the vertical direction is perpendicular to the World Geodetic System (WGS84) ellipsoid model of the Earth's surface, the horizontal plane is perpendicular to the vertical direction, and the rotational directions clockwise and counter-clockwise are determined by a view from above.

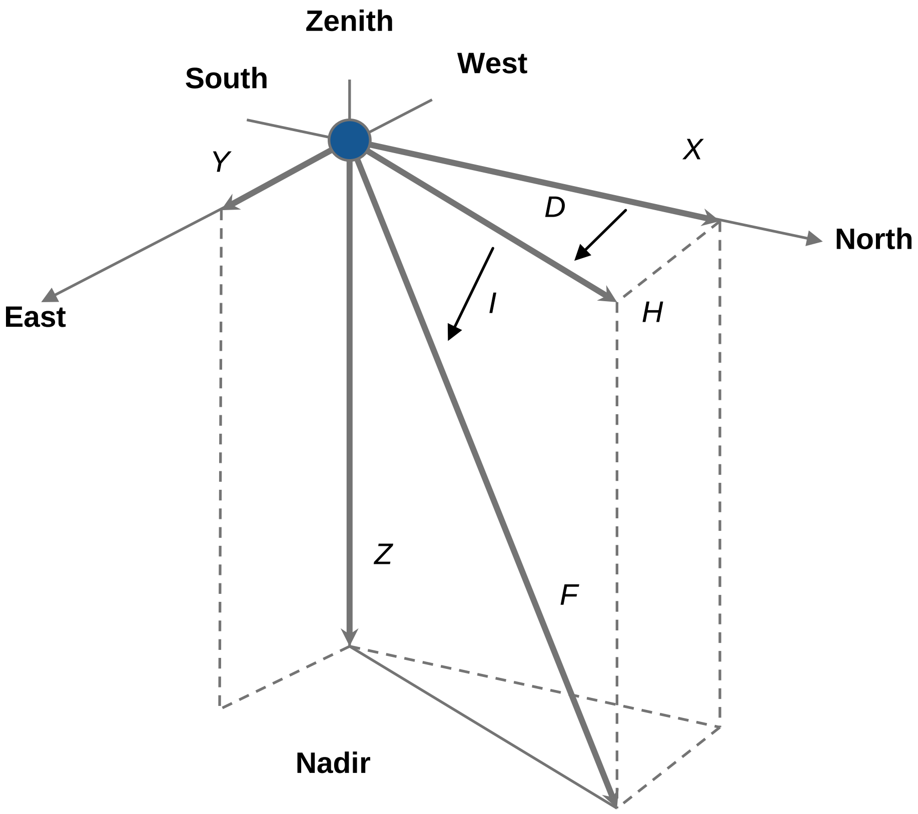

Figure 3: The 7 elements of the geomagnetic field vector **B** in the geodetic reference frame. Here, *F* is the length of the vector **B.**

The quantities *X, Y,* and *Z* are the sizes of perpendicular vectors that add vectorially to **B**; their directions form a local Cartesian coordinate system. The corresponding local spherical polar coordinates are denoted by (*F, I, D*). In both these local coordinate systems, the orientation varies with position on the surface.

The magnetic elements *H, F, I* and *D* are computed from the orthogonal components:

$$H = \sqrt{X^2 + Y^2}, \quad F = \sqrt{H^2 + Z^2}, \quad I = \arctan(Z, H), \quad D = \arctan(Y, X) \tag{4}$$

where $\arctan(a,b)$ is $\tan^{-1}(a/b)$, taking into account the angular quadrant, avoiding a division by zero, and resulting in a declination in the range of $-\pi$ to π and inclination of $-\pi/2$ to $\pi/2$ (In equations, and in computing sub-routines, angles are in radians, but typical user software allows input/output in degrees). Conversely, *X*, *Y*, and *Z* can be determined from the quantities *F*, *I*, and *D.*

The secular variation of these elements is computed using

$$\dot{H} = \frac{X \cdot \dot{X} + Y \cdot \dot{Y}}{H}$$

$$\dot{F} = \frac{X \cdot \dot{X} + Y \cdot \dot{Y} + Z \cdot \dot{Z}}{F}$$

$$\dot{I} = \frac{H \cdot \dot{Z} - Z \cdot \dot{H}}{F^2} \tag{5}$$

$$\dot{D} = \frac{X \cdot \dot{Y} - Y \cdot \dot{X}}{H^2}$$

where $\dot{I}$ and $\dot{D}$, are given in radians/year. These angular changes are typically converted by user software to arc-minutes/year.

3.2.3 Transform of magnetic vector components from geocentric to geodetic frame

The geocentric magnetic field vector components *X´*, *Y´* and *Z´*, are rotated into the geodetic reference frame, using

$$\begin{aligned} X &= X'\cos(\varphi' - \varphi) - Z'\sin(\varphi' - \varphi) \\ Y &= Y' \\ Z &= X'\sin(\varphi' - \varphi) + Z'\cos(\varphi' - \varphi) \end{aligned} \tag{6}$$

with corresponding equations for the time derivatives of the vector components, $\dot{X}'$, $\dot{Y}'$ and $\dot{Z}'$

$$\begin{aligned} \dot{X} &= \dot{X}'\cos(\varphi' - \varphi) - \dot{Z}'\sin(\varphi' - \varphi) \\ \dot{Y} &= \dot{Y}' \\ \dot{Z} &= \dot{X}'\sin(\varphi' - \varphi) + \dot{Z}'\cos(\varphi' - \varphi) \end{aligned} \tag{7}$$

3.3 Specification of the magnetic model

3.3.1 Potential of the magnetic field

In the absence of local electric currents, the magnetic field **B** behaves as a potential field and can be written as the negative spatial gradient of a scalar potential *V*, satisfying the Laplace equation $\nabla \cdot \nabla V = 0$. In geocentric spherical coordinates (λ*,* φ´, *r*) this gives

$$\boldsymbol{B}(\lambda, \varphi', r, t) = -\nabla V(\lambda, \varphi', r, t) \tag{8}$$

The potential V is expanded in terms of spherical harmonics:

$$V(\lambda,\varphi',r,t) = a\sum_{n=1}^{N_i}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}({}_i g_n^m(t)\cos m\lambda + {}_i h_n^m(t)\sin m\lambda)\breve{\mathrm{P}}_n^m(\sin\varphi')$$
$$+ a\sum_{n=1}^{N_e}\left(\frac{r}{a}\right)^{n}\sum_{m=0}^{n}({}_e g_n^m(t)\cos m\lambda + {}_e h_n^m(t)\sin m\lambda)\breve{\mathrm{P}}_n^m(\sin\varphi') \quad (9)$$

where N_i and N_e are the truncation degrees of internal and external expansion, respectively, a (6371200 m) is the geomagnetic reference radius, (λ,φ',r) are the longitude, latitude and radius in a spherical geocentric reference frame, and ${}_i g_n^m(t)$ and ${}_i h_n^m(t)$ are the internal and ${}_e g_n^m(t)$ and ${}_e h_n^m(t)$ the external time-dependent Gauss coefficients of degree *n* and order *m*. The Schmidt semi-normalized associated Legendre functions $\breve{P}_n^m(\mu)$, where $\mu = \sin\varphi'$ is a real number in the interval [-1;1], are defined as

$$\breve{P}_n^m(\mu) = \sqrt{2\frac{(n-m)!}{(n+m)!}}\,P_{n,m}(\mu) \quad if \quad m > 0$$
$$\breve{P}_n^m(\mu) = P_{n,m}(\mu) \quad if \quad m = 0 \quad (10)$$

This follows the definition of $P_{n,m}(\mu)$ commonly used in geodesy and geomagnetism (e.g., Heiskanen and Moritz, 1967; Langel, 1987). Sample functions, for geocentric latitude, φ', are:

$$P_{3,0}(\sin\varphi') = \frac{1}{2}(\sin\varphi')(5\sin^2\varphi' - 3)$$
$$P_{3,1}(\sin\varphi') = -\frac{3}{2}(\cos\varphi')(1 - 5\sin^2\varphi') \quad (11)$$
$$P_{3,2}(\sin\varphi') = 15(\sin\varphi')(1 - \sin^2\varphi')$$
$$P_{3,3}(\sin\varphi') = 15\cos^3\varphi'$$

These $P_{n,m}(\mu)$ are related to the $P_n^m(\mu)$ defined in the NIST Handbook of Mathematical Functions (Section 14.2, page 352) or Gradshteyn and Ryzhik (1994, Chapter 8.7) by $P_{n,m}(\mu) = (-1)^m P_n^m(\mu)$.

3.3.2 Geomagnetic reference radius

The geomagnetic reference radius used in the spherical harmonic expansion of the magnetic potential is fixed by convention at a=6371200 m. The Gauss coefficients are referenced to this value. It is therefore important to use exactly this value in the computation of the field elements.

3.3.3 Epoch of a model

The Gauss coefficients of a geomagnetic reference model are specified in terms of a fixed base-date, t_0, the so-called epoch of the model. The epoch is given in decimal year; for example as *2010.0*.

3.3.4 Validity of a model

Due to non-linear changes of the Earth's magnetic field, geomagnetic reference models are valid only for a limited period. The start and end dates of a model are given in decimal year. For example, a model validity of *2010.0* to *2015.0* means that the model is valid from 00:00:00 *2010-Jan-01* to 00:00:00 *2015-Jan-01*.

The 'external' coefficients represent fields from a source at some region outside r=a. For an observation point further out than the source, the corresponding field is now 'internal'. So there is a limit in radius beyond which the external coefficients are not appropriate. This radius will be specified by the model, along with the maximum degree of the external terms. Magnetic fields caused by electric currents in the magnetosphere are specifically covered by ISO Standard 22009 "Space systems - Model of the Earth's magnetospheric magnetic field".

3.3.5 Time-dependence of Gauss coefficients

The Gauss coefficients ${}_i g_n^m(t)$, ${}_i h_n^m(t)$, ${}_e g_n^m(t)$ and ${}_e h_n^m(t)$ are determined for the desired time *t* from the model coefficients ${}_i g_n^m(t_0)$, ${}_i h_n^m(t_0)$, ${}_e g_n^m(t_0)$ and ${}_e h_n^m(t_0)$, and the linear secular variation model coefficients ${}_i \dot{g}_n^m(t_0)$, ${}_i \dot{h}_n^m(t_0)$, ${}_e \dot{g}_n^m(t_0)$ and ${}_e \dot{h}_n^m(t_0)$, at epoch t_0 as

$$\begin{aligned}
{}_i g_n^m(t) &= {}_i g_n^m(t_0) + (t - t_0)\ {}_i \dot{g}_n^m(t_0) \\
{}_i h_n^m(t) &= {}_i h_n^m(t_0) + (t - t_0)\ {}_i \dot{h}_n^m(t_0) \\
{}_e g_n^m(t) &= {}_e g_n^m(t_0) + (t - t_0)\ {}_e \dot{g}_n^m(t_0) \\
{}_e h_n^m(t) &= {}_e h_n^m(t_0) + (t - t_0)\ {}_e \dot{h}_n^m(t_0)
\end{aligned} \tag{12}$$

Here again, time is given in decimal years and t_0 is the epoch of the model.

3.3.6 Calculation of magnetic vector components in the geocentric reference frame

The magnetic field vector components *X′*, *Y′* and *Z′* in the geocentric reference frame are computed as

$$\begin{aligned}
X'(\lambda, \varphi', r, t) &= -\frac{1}{r}\frac{\partial V}{\partial \varphi'} \\
&= -\sum_{n=1}^{N_i}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}({}_i g_n^m(t)\cos m\lambda + {}_i h_n^m(t)\sin m\lambda)\frac{d\,\breve{\mathrm{P}}_n^m(\sin\varphi')}{d\varphi'} \\
&\quad -\sum_{n=1}^{N_e}\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n}({}_e g_n^m(t)\cos m\lambda + {}_e h_n^m(t)\sin m\lambda)\frac{d\,\breve{\mathrm{P}}_n^m(\sin\varphi')}{d\varphi'}
\end{aligned} \tag{13}$$

$$\begin{aligned}
Y'(\lambda, \varphi', r, t) &= -\frac{1}{r\cos\varphi'}\frac{\partial V}{\partial \lambda} \\
&= \frac{1}{\cos\varphi'}\sum_{n=1}^{N_i}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n} m({}_i g_n^m(t)\sin m\lambda - {}_i h_n^m(t)\cos m\lambda)\,\breve{\mathrm{P}}_n^m(\sin\varphi') \\
&\quad + \frac{1}{\cos\varphi'}\sum_{n=1}^{N_e}\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n} m({}_e g_n^m(t)\sin m\lambda - {}_e h_n^m(t)\cos m\lambda)\,\breve{\mathrm{P}}_n^m(\sin\varphi')
\end{aligned} \tag{14}$$

$$\begin{aligned}
Z'(\lambda, \varphi', r, t) &= \frac{\partial V}{\partial r} \\
&= -\sum_{n=1}^{N_i}(n+1)\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}({}_i g_n^m(t)\cos m\lambda + {}_i h_n^m(t)\sin m\lambda)\,\breve{\mathrm{P}}_n^m(\sin\varphi') \\
&\quad + \sum_{n=1}^{N_e} n\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n}({}_e g_n^m(t)\cos m\lambda + {}_e h_n^m(t)\sin m\lambda)\,\breve{\mathrm{P}}_n^m(\sin\varphi')
\end{aligned} \tag{15}$$

Correspondingly, the secular variation of the magnetic field components are computed as

$$\begin{aligned}\dot{X}'(\lambda,\varphi',r,t_0) &= -\frac{1}{r}\frac{\partial \dot{V}}{\partial \varphi'} \\ &= -\sum_{n=1}^{N_i}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left({}_i\dot{g}_n^m(t_0)\cos m\lambda + {}_i\dot{h}_n^m(t_0)\sin m\lambda\right)\frac{d\,\breve{\mathrm{P}}_n^m(\sin\varphi')}{d\varphi'} \\ &\quad -\sum_{n=1}^{N_e}\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n}\left({}_e\dot{g}_n^m(t)\cos m\lambda + {}_e\dot{h}_n^m(t)\sin m\lambda\right)\frac{d\,\breve{\mathrm{P}}_n^m(\sin\varphi')}{d\varphi'}\end{aligned} \quad (16)$$

$$\begin{aligned}\dot{Y}'(\lambda,\varphi',r,t_0) &= -\frac{1}{r\cos\varphi'}\frac{\partial \dot{V}}{\partial \lambda} \\ &= \frac{1}{\cos\varphi'}\sum_{n=1}^{N_i}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n} m\left({}_i\dot{g}_n^m(t_0)\sin m\lambda - {}_i\dot{h}_n^m(t_0)\cos m\lambda\right)\breve{\mathrm{P}}_n^m(\sin\varphi') \\ &\quad +\frac{1}{\cos\varphi'}\sum_{n=1}^{N_e}\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n} m\left({}_e\dot{g}_n^m(t)\sin m\lambda - {}_e\dot{h}_n^m(t)\cos m\lambda\right)\breve{\mathrm{P}}_n^m(\sin\varphi')\end{aligned} \quad (17)$$

$$\begin{aligned}\dot{Z}'(\lambda,\varphi',r,t_0) &= \frac{\partial \dot{V}}{\partial r} \\ &= -\sum_{n=1}^{N_i}(n+1)\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left({}_i\dot{g}_n^m(t_0)\cos m\lambda + {}_i\dot{h}_n^m(t_0)\sin m\lambda\right)\breve{\mathrm{P}}_n^m(\sin\varphi') \\ &\quad +\sum_{n=1}^{N_e} n\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n}\left({}_e\dot{g}_n^m(t)\cos m\lambda + {}_e\dot{h}_n^m(t)\sin m\lambda\right)\breve{\mathrm{P}}_n^m(\sin\varphi')\end{aligned} \quad (18)$$

3.3.7 Spatial wavelength

A model of spherical harmonic degree N accounts for magnetic fields that have spatial wavelengths larger or equal to $360°/\sqrt{N(N+1)}$ in arc length (see Sec. 3.6.3 of Backus *et al*., 1996).

3.3.8 RMS difference between two models

It is sometimes important to assess the difference between two geomagnetic reference models. These could be successive updates of the same model for different epochs, or two entirely different models. The mean-square of the difference-vector ($\mathbf{B}_1$-$\mathbf{B}_2$) between two models, taken over the sphere at the reference radius *a*, is computed following Backus *et al.* (1996, Sec. 4.4.2) as

$$R(t) = \frac{1}{4\pi}\int_{-\pi/2}^{\pi/2}\int_{0}^{2\pi}\left(\mathbf{B}_1(\lambda,\varphi',a,t) - \mathbf{B}_2(\lambda,\varphi',a,t)\right)^2\cos\varphi'\,d\lambda\,d\varphi' = \sum_{1}^{N} R_n(t) \quad (19)$$

$$R_n(t) = (n+1)\sum_{m=0}^{n}[({}_{1,i}g_n^m(t) - {}_{2,i}g_n^m(t))^2 + ({}_{1,i}h_n^m(t) - {}_{2,i}h_n^m(t))^2)]$$
$$+ n\sum_{m=0}^{n}[({}_{1,e}g_n^m(t) - {}_{2,e}g_n^m(t))^2 + ({}_{1,e}h_n^m(t) - {}_{2,e}h_n^m(t))^2)]$$

Here, $R(t)$ is the mean-square of the difference-vector between two models up to degree *N* at time *t,* the double integral provides the average over the geomagnetic reference sphere, and $R_n(t)$ is the degree-variance of the difference-vector of degree *n*. Here, the Gauss coefficients ${}_{1,i}g_n^m(t)$, ${}_{1,i}h_n^m(t)$, ${}_{1,e}g_n^m(t)$ and ${}_{1,e}h_n^m(t)$ of the first model and the coefficients ${}_{2,i}g_n^m(t)$, ${}_{2,i}h_n^m(t)$, ${}_{2,e}g_n^m(t)$ and ${}_{2,e}h_n^m(t)$ of the second model are assumed zero for *n* larger than the respective degree of the expansion. The RMS difference at time *t* between two models is then defined as

$$d_{RMS}(t) = \sqrt{R(t)} \qquad (20)$$

4 Model usage examples

4.1 Compute reference magnetic vector in near-Earth space

In Low Earth Orbit (LEO) it is conventional to express the magnetic field vector in the geocentric frame. To compute such a reference magnetic field vector, e.g. to determine the attitude of a spacecraft, typically requires the following steps:

1. Determine date *t* in decimal years and position of the spacecraft given in longitude, geocentric latitude and radius
2. Compute the Gauss coefficients ${}_ig_n^m(t)$, ${}_ih_n^m(t)$, ${}_eg_n^m(t)$ and ${}_eh_n^m(t)$ for all degrees *n* and orders *m* for the desired date *t* (Sec 3.3.5)
3. Compute model magnetic field vector components *X´*, *Y´* and *Z´* in geocentric reference frame (Sec 3.3.6), e.g. to compare with measured magnetic field vector to determine attitude of the spacecraft

4.2 Compute reference magnetic elements near the Earth's surface

Near the Earth's surface it is conventional to express the magnetic field vector in the geodetic frame. To compute such a reference magnetic field for a given location and date typically requires the following steps:

1. Choose the desired date *t* in decimal years, and the longitude, geodetic latitude and height above the WGS 84 ellipsoid. Heights above mean sea level (approximately following the equipotential surface known as the geoid) need to be converted to height above the ellipsoid. This can be achieved, for example, by using the online geoid calculator at http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm96/intpt.html, which gives the height of the geoid above the WGS 84 ellipsoid.
2. Convert geodetic to geocentric coordinates (Sec 3.1.3)
3. Compute the Gauss coefficients ${}_ig_n^m(t)$, ${}_ih_n^m(t)$, ${}_eg_n^m(t)$ and ${}_eh_n^m(t)$ for all degrees *n* and orders *m* for the desired date *t* (Sec 3.3.5)
4. Compute magnetic field vector components *X´*, *Y´* and *Z´* in geocentric reference frame (Sec 3.3.6)
5. Since the inclination of the vertical and northward directions differ between the geocentric and geodetic reference frames, the magnetic field vector components have to be rotated from the geocentric to the geodetic reference frame (Sec 3.2.3) to obtain *X*, *Y* and *Z*.
6. Compute the magnetic declination and any other desired magnetic elements (Sec 3.2.2) in the geodetic reference frame.

The computed magnetic field elements in the geodetic reference frame are with respect to a local unit coordinate system where the down axis is perpendicular to the surface of the WGS84 ellipsoid. Deflections of the vertical caused by local gravity anomalies have to be taken care of by the user and are not covered by this standard.

Annex A
(informative)

1 International Geomagnetic Reference Field

The International Geomagnetic Reference Field (IGRF) was introduced by the International Association of Geomagnetism and Aeronomy (IAGA) in 1968 in response to the demand for a standard spherical harmonic representation of the Earth's main field. The model is updated at 5-yearly intervals, produced and released by IAGA Working Group V-MOD.

1.1 Model web site

http://www.ngdc.noaa.gov/IAGA/vmod/

1.2 Stand-alone software

The following public domain software for the IGRF is available for download at the IGRF web site:

Geomag.c: Program in C-language maintained by NOAA's National Geophysical Data Center, distributed both as source code and as precompiled versions for Windows and Linux. It computes the magnetic field vector and elements for given locations (geocentric or geodetic) and dates (or ranges of dates). The program further has command line and spreadsheet options to facilitate repeated processing.

Igrf.f: A FORTRAN program maintained by the British Geological Survey. The IGRF coefficients are embedded into the source code. Options include values at different locations at different times (spot), values at same location at one year intervals (time series), grid of values at one time (grid); geodetic or geocentric coordinates, latitude & longitude entered as decimal degrees or degrees & minutes (not in grid), and choice of main field or secular variation or both (grid only).

1.3 Online calculators

1.3.1 Declination, single point

http://www.ngdc.noaa.gov/geomagmodels/Declination.jsp

http://www-app3.gfz-potsdam.de/Declinationcalc/declinationcalc.html

1.3.2 Magnetic vector and elements, single point

http://www.ngdc.noaa.gov/geomagmodels/IGRFWMM.jsp

http://wdc.kugi.kyoto-u.ac.jp/igrf/point/index.html

http://www.geomag.bgs.ac.uk/gifs/igrf_form.shtml

1.3.3 Magnetic vector and elements, grids and profiles

http://www.ngdc.noaa.gov/geomagmodels/IGRFGrid.jsp

http://omniweb.gsfc.nasa.gov/vitmo/igrf_vitmo.html

1.4 Charts of the magnetic elements

http://wdc.kugi.kyoto-u.ac.jp/igrf/index.html

Glossary of acronyms

IAGA	International Association for Geomagnetism and Aeronomy
WGS84	World Geodetic System 1984